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Considering adjustable production rate in economic optimization for statistical process control and preventive maintenance

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Abstract

Considering the strong correlation between production rate and degradation, we introduce an adjustable production rate into the economic design of statistical process control (SPC) and maintenance. Our goal is to optimize the integrated policy for the X-bar control chart, preventive maintenance, and production rate from an economic standpoint. In this study, we define three policies: production policy, SPC policy, and maintenance policy. We base our analysis on a stochastic process that describes system degradation. A joint optimization model maximizing the earnings rate is established, leveraging a unique recursive method as a consistent tool to systematically derive key parameters and variables required for model formulation. Overall equipment effectiveness (OEE) is quantified to evaluate production performance in complex scenarios. A case study validates the effectiveness of the proposed method, presenting that an appropriate production rate adjustment frequency is crucial to the optimization, that the degradation rate exerts the most significant impact on production rate decisions, and that achieving the maximum earnings rate does not always correspond to the highest OEE.

Keywords: adjustable production rate, statistical process control, preventive maintenance, economic optimization, overall equipment effectiveness (OEE)

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1. Introduction

Quality is a decisive factor for market competition, and it has played a crucial role in many manufacturing companies. As an effective tool for quality improvement, statistical process control (SPC) has been widely implemented in practice. A manufacturing process can be monitored online in real time by using quality control charts (Nenes *et al* 2014, Li *et al* 2021). The alarm will be triggered to signal the occurrence of assignable causes, aiming to reduce the fraction of non-conforming products produced by the system. Meanwhile, as automation increases, equipment must be maintained in good condition to address quality challenges. Improved performance of equipment through either corrective or preventive actions often leads to enhanced product quality (Wang *et al* 2024b)). Both SPC and equipment maintenance are motived by quality control needs, and they are interrelated (Li and Shu 2020, Zhong *et al* 2016). Due to this close relationship, a significant number of studies have been conducted from an economic or economic-statistical perspective on the integrated optimization of control chart and maintenance (Hadidi *et al* 2012, Farahani and Tohidi 2021). A variety of optimization models have been developed to determine the optimal values for sample size, sampling intervals, control limits, and maintenance-related parameters, which can be regarded as a joint policy for quality and maintenance (Cassady *et al* 2000, Pandey *et al* 2010, Hadidi *et al* 2012, Farahani and Tohidi 2021). The renewal reward theory (Ross 2023) is commonly applied in modeling, and the cost rate within the renewal cycle is frequently used as the objective function in most of these optimization models (Pasha *et al* 2020, Rasay *et al* 2022). A few of other objectives are the earnings rate, i.e. earnings per unit time (Panagiotidou and Tagaras 2010, Panagiotidou and Tagaras 2007). There is no essential difference between the two objectives in calculation techniques; the choice of which one to use depends entirely on the specific demands. Therefore, in this study, we use the expected earnings per unit time as our optimization objective.

There have also been a significant number of studies that consider production policy in the jointly economic design of quality and maintenance. Recently, both Hadidi *et al* (2012) and Farahani and Tohidi (2021) review the integrated models in production planning and scheduling, inventory, maintenance, and quality. They point out that these factors are interrelated and need to be considered comprehensively. Literature concerning production in the joint optimization of quality and maintenance primarily focuses on two issues. The first one is production planning, which involves determining economic production quantity Lam and Rahim (2002), Liu *et al* (2017), Wee and Widyadana (2012), Jafari and Makis (2015), Cheng *et al* (2018), inventory level Cheng *et al* (2018), and buffer stocking Lopes (2018), Shi *et al* (2024), Wang *et al* (2025). The second issue is production scheduling, which focuses on determining the quantities and sequences of tasks Farahani and Tohidi (2021). However, another production issue, i.e. production rate (number of outputs per unit time), has not been considered in the joint optimization of quality and maintenance. Literature has indicated that the system generally deteriorates more quickly at higher production rates Sun *et al* (2019), Uit het Broek *et al* (2020), Uit het Broek *et al* (2021), Hu *et al* (2022), Li *et al* (2023), Vlastos *et al* (2023), Li *et al* (2025). The production rate is much related to the degradation, and the relationships between them have been given by Sun *et al* (2019), Uit het Broek *et al* (2021), Hu *et al* (2022), and Sun *et al* (2023). The production rate's direct influence on system degradation creates an unavoidable inter-dependency between process control and maintenance. By integrating production rate dynamics into their joint optimization, organizations can develop adaptive, resilient systems that maintain high productivity while mitigating degradation-related risks, ultimately enhancing long-term operational sustainability and competitive advantage in complex engineering environments. Therefore, it is essential to consider the production rate in the joint optimization of SPC and maintenance.

Because higher production rates lead to faster system degradation, the degradation rate is dependent on the production rate. Controlling system degradation by selecting an appropriate production rate is common in real production. Therefore, the mutual influence between degradation and production rate is a prerequisite for production rate decision-making. There has been extensive research on modeling the degradation process itself, including physical models and data-driven models. Data-driven models are favored by many scholars because they do not require an understanding of degradation mechanisms. These models mainly include stochastic process models and machine learning models. For stochastic process models for degradation modeling, the Wiener process and gamma process are two of the most commonly used methods Lin *et al* (2021), Wang *et al* (2024a). Currently, only references Uithet Broek *et al* (2020, 2021) have considered linking the system degradation with production rates, but they only focus on the gamma process and lack research on other stochastic processes. Regarding the relationship between degradation and production rate, some scholars have already conducted relevant work. For example, Sun *et al* (2023) have modeled the relationship between degradation rate and production rate, while Francie *et al* (2014) and Li *et al* (2023) have established models relating failure rate to the production rate. However, these studies have not considered the relationship between the production rate and stochastic degradation processes. Therefore, this paper establishes a relationship model between stochastic processes and the production rate, using mathematical expressions to link the degradation process with the production rate, thereby providing theoretical support for production rate decision-making.

Another issue we need to focus on is that the joint policy of production, SPC, and maintenance is closely related to overall equipment effectiveness (OEE). As we know, OEE a critical metric used to assess the production efficiency by considering three main factors: availability, performance, and quality. The performance is related to the mean production rate, availability is associated with maintenance duration, and quality is

determined by the effectiveness of process control Schiraldi and Varisco (2020), Mendonça *et al* (2022). By evaluating OEE, we can determine whether companies are achieving significant economic benefits; therefore, it is worth analyzing the influence of the joint policy on the OEE, which has not been studied yet.

Our aim is to find an economically joint policy for SPC, maintenance, and production. In this study, we define three policies: production policy, SPC policy, and maintenance policy. A stochastic process for describing system degradation, as used in Li *et al* (2025), serves as the basis for our analysis. An optimization model for the earnings rate is established based on the derivations for the expected duration of a renewal cycle, expected process control cost, expected quality loss, and expected sales income. Additionally, the OEE for the studied scenario is defined and calculated. Our main contributions are as follows.

- Since degradation relies on the production rate, the parameters in the stochastic processes are expressed as functions of both the degradation rate and the production rate. This study establishes a general relationship between the degradation process and the production rate, which further aids in decision-making regarding production rates.
- A recursive method is introduced to obtain various intermediate variables necessary for constructing the optimization model. This method is based on the four potential scenarios that emerge after each inspection using a control chart.
- With performance determined by the production rate, and availability and quality defined based on the process states, the OEE for this complex production system is defined and calculated, providing an effective indicator for evaluating production efficiency.

The remainder is organized as follows. Section 2 presents problem with three policies described in details. In section 3, we formulate mathematical expressions necessary for the problem. Section 4 includes the case study. Finally, in section 5, we summarize our findings and provide future research suggestions.

2. Problem statement

A single-unit production system that begins operating from a brand-new state is considered. The system degradation is characterized by a stochastic process, with unobservable degradation states. A single quality characteristic of the products is assumed to be both observable and measurable, serving to reflect system conditions. An X-bar control chart closely monitors the statistics of this quality characteristic, providing the foundation for assessing whether the production process is functioning within control limits or has deviated from them. Both preventive maintenance and corrective maintenance are perfect; in other words, they can restore the system into a like-new condition after completion. Thus, the maintenance functions as a renewal process for the system. During the production process, the production rate can be adjusted when necessary, depending on whether the adjustment will provide benefits. The decision variables and parameters that will be used in this study are listed as follows.

Decision variables:

v_k	kth production rate
l_k	Number of samplings during which the system operates at v_k
K	Coefficient of the control limits for the X-bar control chart
n	Sample size
d_1	Duration of the first sampling interval

Parameters:

J	Number of sampling intervals prior to implementing PM
u_j	Production rate during the jth sampling interval
K	Number of production rates
$X(t)$	Degradation level at time t
x	Degradation rate
$f_u(x)$	Probability density function of the degradation rate under production rate u
θ	Shape parameter of the gamma density function
β	Scale parameter of the gamma density function
x_0	Initial degradation level at the starting moment in a Wiener process
μ	Drift parameter of the Wiener process
σ	Diffusion parameter of the Wiener process
$B(\cdot)$	Standard Brownian motion
$E(x\|u)$	Mean of the degradation rate under production rate u
$\lambda(u)$	Mean degradation rate function with respect to production rate u
$\mathrm{Var}(x\|u)$	Variance of the degradation rate under production rate u
$\sigma\mathrm{max}$	Standard deviation of the degradation rate under the maximum production rate
δ	Coefficient of variation of the degradation rate
ω	Exponential term in the function $\lambda(u)$
$\lambda_{\min}$	Minimum value of the deterioration rate
$\lambda_{\max}$	Maximum value of the deterioration rate
L	Threshold of deterioration level for system failure
M	Number of the states
Δx	Step size for the discretization of deterioration level
$F_u(Y)$	Cumulative distribution of the degradation rate at production rate u
$\boldsymbol{P}(u)$	State transition probability matrix of the degradation under production rate u
$\boldsymbol{\pi}(0)$	Initial probability distribution
t_j	Time at the end of the jth sampling interval
$\boldsymbol{\pi}(t_j)$	Probability distribution at the end of the jth sampling interval
$\boldsymbol{\pi}_M(t_j)$	Probability of the system fails at t_j
D_z	Number of the unit times in zth sampling interval
$t_{j,i}$	Time at the end of the ith time unit in the jth sampling interval
μ_0	Mean of the normal distribution in the in-control process

σ_0	Standard deviation of the normal distribution in the out-of-control process
η	Quality shift parameter
a	Probability of the first type of error
b	Probability of the second type of error
C_I	Average cost of the identification for a false alarm
c_f	Fixed cost for each sampling
c_v	Cost for each additional sample
T_{pm}	Average duration of a PM
C_{pm}	Average cost of a PM
T_{cm}	Average duration of a CM
C_{cm}	Average cost of a CM
T_c	Duration of the PM cycle
p_j	Probability that the system fails at time t_j, assuming it was in-control at time t_{j-1}
τ_j	Expected in-control duration in the jth sampling interval, assuming system failure occurs during this interval
N_j	Expected number of samplings after time t_j, assuming the process is out-of-control at time t_j without triggering a true alarm
$E(T)$	Expected duration of a renewal cycle
$E(t^{IC})$	Expected duration of being in control during the renewal cycle
$E(t^{OC})$	Expected duration of being out of control during the renewal cycle
$E(t^{pm})$	Expected PM duration during the renewal cycle
$E(t^{cm})$	Expected CM duration during the renewal cycle
$E(H)$	Expected process control cost throughout the entire renewal cycle
$E(Q)$	Quality loss produced throughout the entire renewal cycle
γ_0	Quality loss per unit time produced in the in-control process, assuming the system runs at production rate of 1
γ_1	Quality loss per unit time produced in the out-of-control process, assuming the system runs at production rate of 1
$E(R)$	Expected sales income over the entire renewal cycle
r	Sales income per unit time produced in the in-control process, assuming the system runs at production rate of 1
$E(e)$	Expected earnings per unit time
Avail	Availability-the proportion of the expected run time to the expected duration of the renewal cycle
Perf	Performance-the average production rate during the entire renewal cycle
Qual	Quality-the ratio of the expected quantity produced during the in-control process to the expected total quantity during the entire renewal cycle
I^{IC}	Expected quantity produced during the in-control process
I^{OC}	Expected quantity produced during the out-of-control process
$E(OEE)$	Expected OEE

2.1. Production policy

The production rate is defined as the quantity of products per unit time. It is standardized within the range from 0 to 1 to facilitate modeling. A minimum value of 0 indicates that the system is unused, while a maximum value of 1 signifies that the system is operating at its maximum capacity. Because the production rate affects degradation, we may opt for the system to operate at a production rate lower than 1 to better control degradation. Therefore, the feasible production rates constitute a set $U = [0,1]$. Let J be the number of sampling intervals prior to implementing PM. We denote the production rate during the jth sampling interval as $u_j \in U$ for $j = 1,\cdots,J$.

In practice, adjusting the production rate is a common approach to controlling system degradation. However, the adjustment of the production rate may incur various potential costs; therefore, we do not make frequent adjustments, performing them only at a few specific sampling moments. Thus, it is reasonable to assume only a limited number of production rates during the production process. Let K represent the number of different production rates, $v_k \in U$ represent the kth production rate for $k = 1,\cdots,K$, $l_k \in \{1,\cdots,J\}$ represent the number of samplings during which the system operates at v_k. We then define the production policy as follows.

(Production policy). For a given number of production rates K, the production policy is defined as $\{v_1,\cdots,v_K,l_1,\cdots,l_K\}$.

It is also obvious that J can be derived from $l_1,l_2,\cdots,l_K$, i.e. $\sum_{k=1}^{K} l_k = J$. The relationship between u_j for $j = 1,\cdots,J$ and v_k for $k \in \{1,2,\cdots,K\}$ is given by $u_j = v_k$. The schematic diagram illustrating this relationship is shown in figure 1. It can be seen in this figure that $u_1 = u_2 = v_1$, $u_3 = v_2$, $u_4 = u_5 = u_6 = v_3$, $\cdots$, $u_J = v_K$. It is noteworthy that the production rate remains consistent across consecutive sampling intervals, indicating that adjustments to the production rate do not occur at each sampling point.

Given the production rate's direct impact on system degradation, the production policy aims to balance production efficiency and degradation control by dynamically adjusting the rate during the production cycle. The production rate will be optimized within the proposed optimization model.

2.2. Degradation process

The degradation process is commonly defined as a continuous-time, continuous-state stochastic process that does not decrease. Two predominant stochastic processes used for modeling degradation are the gamma process and the Wiener process.

(i) Gamma process. Let $\{X(t)\,|t\rangle\,0\}$ be the degradation level at time t, which is a gamma process. The degradation increments are independently gamma distributed in a gamma process. Therefore, the density function of the degradation rate x can be written as:

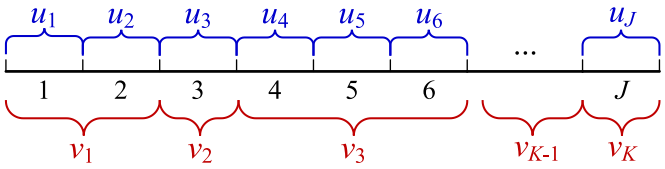


Figure 1. Schematic diagram for the relationship between u_j and v_k.

$$f_u(x) = \frac{x^{\theta-1}\exp(-x/\beta)}{\Gamma(\theta)\beta^{\theta}} \tag{1}$$

where $\theta > 0$ is the shape parameter, $\beta > 0$ is the scale parameter, and $\Gamma(\theta) = \int_0^{\infty} z^{\theta-1}\exp(-z)\,\mathrm{d}z$ is the gamma function. The mean of the degradation rate under production rate u is $E(x|u) = \theta\beta$, and the variance is $\mathrm{Var}(x|u) = \theta\beta^2$.

(ii) Wiener process. The Wiener process $X(t) = x_0 + \mu t + \sigma B(t)$ is also often used for modeling the stochastic degradation process, where x_0 represents the initial degradation level at the starting moment (usually), $\mu > 0$ is the drift parameter, σ is the diffusion parameter, and $B(\cdot)$ represents the standard Brownian motion that depicts the stochastic dynamics of the degradation process. The degradation rate x in the Wiener process follows the normal distribution with the probability density function:

$$f_u(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right). \tag{2}$$

The mean of the degradation rate under production rate u is $E(x|u) = \mu$, and the variance is $\mathrm{Var}(x|u) = \sigma^2$.

(Properties of the degradation rate). Some properties of the gamma and Wiener degradation processes under the production rate u are provided as follows.

(i) The mean of the degradation rate under the production rate u is $E(x|u) = \lambda(u)$, where $\lambda(u)$ is defined in the equation (3).
(ii) The variance of the degradation rate reaches its maximum, denoted as $\sigma^2_{\max}$, under the maximum production rate of 1, i.e. $\mathrm{Var}(x|u=1) = \sigma^2_{\max}$.
(iii) The coefficient of variation of the deterioration rate remains unaffected by the production rate, i.e. $\delta = \frac{\sigma}{\mu}$.

Similar to Li *et al* (2025), Uithet Broek *et al* (2020), and Sun *et al* (2023), the connection between the production rate and the degradation rate can be defined as follows.

$$\lambda(u) = \lambda_{\min} + (\lambda_{\max} - \lambda_{\min})\,u^{\omega} \tag{3}$$

where $\lambda_{\min}$ is the average degradation rate for the unused system, i.e. $\lambda_{\min} = \lambda(0)$, while $\lambda_{\max}$ is the average degradation rate for the system that operates at its maximum production rate, i.e. $\lambda_{\max} = \lambda(1)$. As $\lambda_{\max} > \lambda_{\min}$, $\lambda(u)$ is an increasing function of u, implying that the system degrades more quickly at higher production rates.

Traditional degradation models often treat degradation as a time-dependent stochastic process with fixed parameters, implicitly assuming that operational conditions, such as production rate, are constant or irrelevant. In contrast, we recognize that in many real-world systems (e.g., manufacturing equipment, energy systems, or battery usage), higher production rates typically accelerate wear and tear, thereby influencing the degradation trajectory. Therefore, we leverage Proposition 1 to explicitly capture the correlation between production rate and degradation by modeling the parameters of the stochastic process as functions of production-rate–related variables.

Proposition 1. For the above two types of degradation processes, their respective parameters have the following relationships with the production rate.

(i) For the gamma process, $\theta = \frac{\lambda^2_{\max}}{\sigma^2_{\max}}$ and $\beta = \lambda(u)\frac{\sigma^2_{\max}}{\lambda^2_{\max}}$,
(ii) For the Wiener process, $\mu = \lambda(u)$ and $\sigma = \lambda(u)\frac{\sigma_{\max}}{\lambda_{\max}}$.

Please see the proof in the appendix. According to the proposition 1, the two types of stochastic processes can not only describe the degradation process but also be related to the production rate. Other stochastic processes are also used for describing system degradation. Similar to the gamma process and Wiener process, as long as their respective parameters have connections with the production rate, our proposed method is applicable to these degradation situations.

The minimum of the degradation level is defined as 0, which indicates a brand-new system. The threshold of the degradation level for system failure is defined as L, which indicates that once the degradation level surpasses L, the system is judged to have failed. In order to facilitate modeling, discretizing a continuous degradation process is a common approach. In this study, the degradation levels are discretized into M intervals. The range of the degradation levels for the ith interval, $i = 1, \cdots, M-1$, is $[(i-1)\Delta x, i\Delta x)$, where $\Delta x = L/(M-1)$, and for the Mth interval is $[L, \infty)$. Clearly, any degradation levels that exceed the threshold L are classified into the Mth interval.

All the degradation levels within the same interval are labeled as a same state; therefore, the system comprises M distinct states. The cumulative distribution function of the degradation rate x at the production rate u is denoted as $F_u(Y)$. For any non-failure state, $i < M$, we define the probability for a state to remain in itself as $F_u(0.5\Delta x)$, and for the transition from any state to another non-failure state $i+s$ as $F_u((s+0.5)\Delta x) - F_u((s-0.5)\Delta x)$. The probability for the transition from any state to the failure state is denoted as $1 - F_u((s-0.5)\Delta x)$. Evidently, $p_u(M,M) = 1 - F_u(-0.5\Delta x) = 1$, implying that once the system enters the failure state, it cannot move to any

other state without an external invention. In conclusion, the state transition matrix for the degradation is defined as follows.

(State transition matrix). The state transition matrix of the degradation associated with u, $P(u)$, is

$$P(u)=\begin{bmatrix} F_u(0.5\Delta x) & F_u(1.5\Delta x)-F_u(0.5\Delta x) & \cdots & F_u((M-1.5)\Delta x)-F_u((M-2.5)\Delta x) & 1-F_u((M-1.5)\Delta x) \\ 0 & F_u(0.5\Delta x) & \cdots & F_u((M-2.5)\Delta x)-F_u((M-3.5)\Delta x) & 1-F_u((M-2.5)\Delta x) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & F_u(0.5\Delta x) & 1-F_u(0.5\Delta x) \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}. \tag{4}$$

The probability distribution at the end of the jth sampling interval, t_j, is

$$\pi(t_j)=\pi(0)\prod_{z=1}^{j}P^{D_z}(u_z) \tag{5}$$

where $\pi(t_j)=\left[\pi_1(t_j),\cdots,\pi_M(t_j)\right]\pi(0)=\left[\pi_1(0),\cdots,\right.$ $\left.,\pi_M(0)\right]$ is the initial probability distribution, and D_z is the quantity of unit times in the zth sampling interval, indicating the duration of this interval.

Based on the Chapman–Kolmogorov equations, it is not difficult to understand the equation (5). Note that D_z for $z=1,\cdots,J$ can be determined by using the method in the section 2.3. Obviously, $\pi_M(t_j)$ is the probability of system failure at t_j.

The probability distribution at the end of the ith time unit in the jth sampling interval, $t_{j,i}$, can be developed as:

$$\pi(t_{j,i})=\pi(0)\left(\prod_{z=1}^{j-1}P^{D_z}(u_z)\right)P^{i}(u_j). \tag{6}$$

It is also easy to understand equation (6) based on the Chapman–Kolmogorov equations.

2.3. Process control policy

In SPC, we start with sampling, followed by the inspections using an X-bar control chart, and then evaluate the necessity of maintenance actions at regular intervals.

(SPC policy). Let k represent the coefficient of the control limits for the X-bar control chart, n represent the sample size, and d_1 represent the duration of the first sampling interval. The SPC policy is defined as $\{k,n,d_1\}$.

The SPC policy aims to timely detect process shifts caused by system degradation or other disturbances, ensuring stable production quality. These three parameters will be optimized within the proposed optimization model to achieve a balanced trade-off between detection efficiency and operational cost.

The rule that the failure probability remains consistent across all sampling intervals is employed to specify the duration of each interval, d_j, i.e.

$$\frac{\pi_M(t_j)-\pi_M(t_{j-1})}{1-\pi_M(t_{j-1})}=\frac{\pi_M(t_1)-\pi_M(0)}{1-\pi_M(0)} \tag{7}$$

where $\pi_M(t)$ is the probability of system failure at time t, and $t=0,t_1,\cdots,t_J$. $t_1=d_1$, and for $j=2,\cdots,J$, $t_j=t_{j-1}+d_j$. By the equation (7), we can determine the duration for each sampling interval. This rule aligns with the 'average risk' principle in reliability engineering. In many practical scenarios, system degradation is a gradual process, and short-term fluctuations in failure probability (e.g., minor production rate spikes) do not significantly alter the long-term reliability profile. By assuming constant failure probability across sampling intervals, the model focuses on mitigating average failure risks over the operational life-cycle, rather than reacting to short-term noise. This approach ensures that the joint strategy of SPC and maintenance is robust to transient operational variations, avoiding over-engineering or under-protection due to temporary changes in system conditions.

An X-bar control chart is used to monitor the mean of the quality characteristic and determine whether the process is in control. When the process is under control, it is assumed that the quality characteristic follows a normal distribution with a mean μ_0 and a standard deviation σ_0. However, if a system failure occurs, the mean may shift from μ_0 to $\mu_0+\eta\sigma_0$, where $\eta>0$, while the standard deviation keeps unchanged. The sample size is n. Typically, if the mean of n samples is within the control limits, no alarm is triggered, indicating the process is in control. If it falls outside, an alarm signals the process is out of control. However, two errors can arise in X-bar control chart monitoring: in control with a false alarm and out of control with no alarm, and their respective probabilities are outlined below.

$$\begin{aligned} a&=2\left[1-\Phi(k)\right], \\ b&=\Phi\left(k-\eta\sqrt{n}\right)-\Phi\left(-k-\eta\sqrt{n}\right). \end{aligned} \tag{8}$$

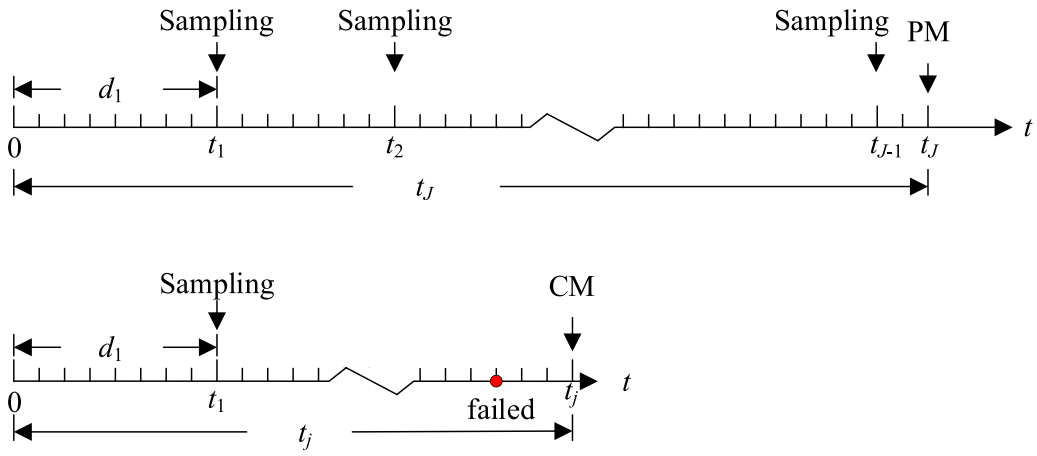


Figure 2. Two potential scenarios in process control.

Therefore, when an alarm occurs, if it is true, a CM will be performed to restore the system to a brand-new state, if it is false, the system will go on, incurring average cost C_{I} for identifying the false alarm. If no alarms arise during the PM cycle, a PM will be performed when time t_J is up. Figure 2 depicts the two potential scenarios of process control.

c_{f} denotes the fixed cost associated with each sampling, while c_{v} represents the variable cost, which is the cost for each additional sample. A PM requires an average time of T_{pm} and an average cost of C_{pm}, while a CM takes an average time of T_{cm} and incurs an average cost of C_{cm}. The cost of each sampling is lower than that of either a CM or a PM. Since an unplanned system failure typically incurs higher costs for the company than a system that is retired as scheduled, it is acceptable to assume that C_{cm} is not lower than C_{pm}, and that the cost per unit time of a CM is not lower than that of a PM. In summary, the various costs in our research must adhere to the following conditions: $0 \leqslant c_{\mathrm{f}} + c_{\mathrm{v}} n \leqslant C_{\mathrm{pm}} \leqslant C_{\mathrm{cm}}$, and $\frac{C_{\mathrm{pm}}}{T_{\mathrm{pm}}} \leqslant \frac{C_{\mathrm{cm}}}{T_{\mathrm{cm}}}$.

(PM policy). For a given $\{d_1, d_2, \cdots, d_J\}$, the PM policy is defined as $T_{\mathrm{c}} = \sum_{j=1}^{J} d_j$, which represents the duration of the PM cycle.

From the definition of the PM policy, we can see that once the production policy and the SPC policy are established, the PM policy is subsequently determined. Likewise, the parameter d_1 in the PM policy will be optimized within the proposed optimized model.

3. Model formulation

We will construct an optimization model to obtain the joint policy for the production rate, X-bar control chart, and preventive maintenance. The objective is to maximize the earnings rate. Therefore, we first formulate the expected earnings rate by developing the equations for the expected duration of a renewal cycle, process control cost, quality loss and sales income, then further construct the optimization model for the optimal joint policy.

Before modeling, we first define and calculate some intermediate parameters.

Let p_j denote the probability of system failure at time t_j, given that it was under control at time t_{j-1}. p_j can be derived by

$$p_j = \frac{\pi_M(t_j) - \pi_M(t_{j-1})}{1 - \pi_M(t_{j-1})}, j = 1, \cdots, J. \tag{9}$$

Let τ_j denote the expected duration of the in-control process during the jth sampling interval, assuming system failure occurs in this interval, then for $j = 1, \cdots, J$, τ_j can be expressed as

$$\tau_j = \frac{\sum_{i=1}^{D_j} i\left(\pi_M(t_{j-1,i}) - \pi_M(t_{j-1,i-1})\right)}{\pi_M(t_j) - \pi_M(t_{j-1})} \tag{10}$$

where $\pi_M(t_{j-1,i})$ is the probability that the system fails at time $t_{j-1} + i\Delta t$, in which Δt is the unit time.

Let N_j represent the expected number of samplings after time t_j, under the assumption that the process is out-of-control at t_j without activating a true alarm. Without consideration of the state of the process at the end of the Jth sampling interval, t_J, the system must be paused for PM, making sampling unnecessary this time.

$N_J = 0$, $N_{J-1} = 0$, $N_{J-2} = 1$, and for $j = 1, \cdots, J-3$, N_j is expressed by

$$N_j = (1-b) \sum_{i=1}^{i=J-2-j} i b^{i-1} + (J-1-j)\, b^{J-2-j}. \tag{11}$$

The probability of sampling i times after time t_j is $b^{i-1}(1-b)$, equation (11) can be easily derived by accumulating all possible scenarios.

3.1. Expected duration of a renewal cycle

Proposition 2. The expected duration of a renewal cycle comprises the expected in-control duration, out-of-control duration, downtime duration (CM duration and PM duration), which can be expressed as

$$E(T)=\sum_{j=1}^{j=J} d_j \prod_{i=1}^{i=j}(1-p_i) + \sum_{j=1}^{j=J} p_j \tau_j \prod_{i=0}^{i=j-1}(1-p_i) + \sum_{j=1}^{j=J} p_j (d_j-\tau_j) \prod_{i=0}^{i=j-1}(1-p_i) + b\sum_{j=1}^{j=J-1} p_j E(V_j) \prod_{i=0}^{i=j-1}(1-p_i) + \prod_{j=1}^{j=J}(1-p_j) T_{\mathrm{pm}} + T_{\mathrm{cm}} \sum_{j=1}^{j=J} p_j \prod_{i=0}^{i=j-1}(1-p_i) \quad (12)$$

where $E(V_j)=\sum_{=j+1}^{i=J} b^{i-j-1} d_i$.

Please see the proof in the appendix, where a recursive method is proposed.

According to the definitions of d_j and τ_j, we can see that $d_j-\tau_j$ represents the out-of-control duration in the jth sampling interval, under the assumption that system failure occurs during this interval. Let $E\left(t^{\mathrm{IC}}\right)$ represent the expected duration of being in control during the renewal cycle, $E\left(t^{\mathrm{OC}}\right)$ represent the expected duration of being out of control, $E(t^{\mathrm{cm}})$ represent the expected duration of CM, and $E(t^{\mathrm{pm}})$ represent the expected duration of PM, thus, $E(T)$ can be represented as the total of these four components, i.e. $E(T)=E\left(t^{\mathrm{IC}}\right)+E\left(t^{\mathrm{OC}}\right)+E(t^{\mathrm{cm}})+E(t^{\mathrm{pm}})$. Clearly, $E\left(t^{\mathrm{IC}}\right)=\sum_{j=1}^{j=J} d_j \prod_{i=1}^{i=j}(1-p_i)+\sum_{j=1}^{j=J} p_j\tau_j \prod_{i=0}^{i=j-1}(1-p_i)$, $E\left(t^{\mathrm{OC}}\right)=\sum_{j=1}^{j=J} p_j(d_j-\tau_j)\prod_{i=0}^{i=j-1}(1-p_i)+b\sum_{j=1}^{j=J-1} p_j E(V_j)\prod_{i=0}^{i=j-1}(1-p_i)$, $E(t^{\mathrm{pm}})=\prod_{j=1}^{j=J}(1-p_j)T_{\mathrm{pm}}$, and $E(t^{\mathrm{cm}})=T_{\mathrm{cm}}\sum_{j=1}^{j=J} p_j \prod_{i=0}^{i=j-1}(1-p_i)$.

3.2. Expected process control cost

SPC and maintenance can be considered as the specific process control measures. Therefore, the expected process control cost over the entire renewal cycle, denoted as $E(H)$, includes both SPC and maintenance costs.

Proposition 3. The expected process control cost throughout the entire renewal cycle can be written as

$$E(H)=(c_{\mathrm{f}}+c_{\mathrm{v}}n)\sum_{j=1}^{j=J-1}\prod_{i=0}^{i=j-1}(1-p_i)+aC_{\mathrm{I}}\sum_{j=1}^{j=J-1}\prod_{i=1}^{i=j}(1-p_i) + b(c_{\mathrm{f}}+c_{\mathrm{v}}n)\sum_{j=1}^{j=J-1} p_j N_j \prod_{i=0}^{i=j-1}(1-p_i) + C_{\mathrm{cm}}\sum_{j=1}^{j=J} p_j \prod_{i=0}^{i=j-1}(1-p_i)+C_{\mathrm{pm}}\prod_{j=1}^{j=J}(1-p_j). \quad (13)$$

Please see the proof of $E(H)$ in the appendix, where the recursive method is also used.

According to the equation (13), when $J=1$, it implies there is no samplings, leading to $E(H)=C_{\mathrm{cm}}\sum_{j=1}^{j=J} p_j \prod_{i=0}^{i=j-1}(1-p_i)+C_{\mathrm{pm}}\prod_{j=1}^{j=J}(1-p_j)$.

3.3. Expected quality loss

The quality loss is characterized as the loss that can occur in both in-control and out-of-control processes. Let γ_0 and γ_1 represent the quality loss per unit time produced in the in-control and out-of-control processes, respectively, assuming that the system runs at the maximum production rate.

Proposition 4. The quality loss produced throughout the entire renewal cycle, denoted as $E(Q)$, can be expressed by

$$E(Q)=\gamma_0\sum_{j=1}^{j=J} u_j d_j \prod_{i=1}^{i=j}(1-p_i)+\gamma_0\sum_{j=1}^{j=J} p_j u_j \tau_j \prod_{i=0}^{i=j-1}(1-p_i) + \gamma_1\sum_{j=1}^{j=J} p_j u_j (d_j-\tau_j)\prod_{i=0}^{i=j-1}(1-p_i) + b\sum_{j=1}^{j=J-1} p_j E(Z_j)\prod_{i=0}^{i=j-1}(1-p_i) \quad (14)$$

where $E(Z_j)=\gamma_1\sum_{i=j+1}^{i=J} b^{i-j-1}u_i d_i$

Please see the proof of $E(Q)$ in the appendix. It is clear that the expected quality loss is divided into two parts: one from the in-control process and the other from the out-of-control process. Clearly, $E(Q)$ is related to the production rates closely.

3.4. Expected sales income

Sales income denotes the income generated from the sales of products manufactured by the in-control process. Let r represent the sales income per unit time, assuming that the system runs at the maximum production rate.

Proposition 5. The expected sales income throughout the entire renewal cycle, denoted as $E(R)$, can be expressed by

$$E(R)=r\sum_{j=1}^{j=J} u_j d_j \prod_{i=1}^{i=j}(1-p_i)+r\sum_{j=1}^{j=J} p_j u_j \tau_j \prod_{i=0}^{i=j-1}(1-p_i). \quad (15)$$

Please see the proof of $E(R)$ in the appendix. $E(R)$ is also related to the production rates.

3.5. Optimization model

The maintenance for the system can be described as a renewal process, during which we pursue the goal of maximizing the expected earnings per unit time, $E(e)$, which is expressed as

$$E(e)=\frac{E(R)-E(H)-E(Q)}{E(T)}. \quad (16)$$

A sample size of 30 is generally considered sufficient to represent population information. Consequently, we determine the optimal sample size within the range of 1–30. To ensure effective monitoring using the X-bar chart, the coefficient of

the control limits is established to be less than 4 (Li *et al* (2025)). The optimization model is constructed under these constraints, as follows.

$$\arg \max_{v_1,\cdots,v_K,l_1,\cdots,l_K,k,n,d_1} E(e). \tag{17}$$

For a given K, the genetic algorithm can be used to solve this model. By using the earnings rate $E(e)$ as the fitness function, we can solve the equation (17) to identify the optimal solution for various decision variables, i.e. $v_1^*,\cdots,v_K^*,l_1^*,\cdots,l_K^*,k^*,n^*,d_1^*$. The steps are as follows.

Step 1. Determine the number of chromosomes and generate an initial population of binary strings.

Step 2. Evaluate the fitness of each individual using the profit rate $E(P)$ as the fitness function.

Step 3. Select individuals for reproduction using roulette wheel selection. Negative fitness values are adjusted to small positive values, the order of individuals is randomized, and the best individual is retained.

Step 4. Combine genetic information from two individuals to create new ones by exchanging parts of their binary strings, while also retaining the best individual from the previous step.

Step 5. Introduce random mutations in the new individuals to maintain genetic diversity, with a low probability of mutation. The best individual also participates in this process.

Step 6. Calculate the fitness of the final individuals and select the one with the highest fitness as the best.

Step 7. Repeat Steps 3–6 until a satisfactory solution is found, a specified number of generations is reached, or improvements fall below a threshold.

3.6. OEE formulation

OEE considers three essential factors: availability, performance, and quality, with each expressed as a percentage, and the OEE is the product of these three percentages. The availability can be determined by calculating the proportion of the expected runtime to the expected duration of the renewal cycle, that is:

$$\text{Avail} = \frac{t^{\text{IC}} + t^{\text{OC}}}{E(T)} \times 100\% \tag{18}$$

Avail reflects the proportion of scheduled production time during which the equipment is actually operational. It accounts for losses due to unplanned downtime and planned stops.

The performance can be defined as the average production rate during the renewal cycle, i.e.

$$\text{Perf} = \frac{\sum_{k=1}^{K} v_k \sum_{i=\sum_{j=1}^{k-1} l_j+1}^{\sum_{j=1}^{k} l_j} d_i}{\sum_{j=1}^{J} d_j} \times 100\%. \tag{19}$$

Since $v_1 = v_2 = \cdots = v_K = v$, we have $\sum_{k=1}^{K} v_k \sum_{i=\sum_{j=1}^{k-1} l_j+1}^{\sum_{j=1}^{k} l_j} d_i = v \sum_{k=1}^{K} \sum_{i=\sum_{j=1}^{k-1} l_j+1}^{\sum_{j=1}^{k} l_j} d_i = v\big(\sum_{i=1}^{l_1} d_i + \sum_{i=l_1+1}^{l_1+l_2} d_i + \cdots + \sum_{i=l_1+l_2+\cdots+l_{K-1}+1}^{l_1+l_2+\cdots+l_K} d_i\big) = v \sum_{j=1}^{J} d_j$; thus we obtain $\text{Perf} = v$. Perf measures how efficiently the equipment operates when running, compared to its ideal or designed cycle time. It captures speed losses such as minor stoppages or operating at reduced speed.

We indicate the quality by the ratio of the expected quantity produced during the in-control time to the expected total quantity during the renewal cycle. Based on the expected duration of the renewal cycle in equation (12), we can derive the quality as

$$\text{Qual} = \frac{I^{\text{IC}}}{I^{\text{IC}} + I^{\text{OC}}} \times 100\% \tag{20}$$

where $I^{\text{IC}} = \sum_{j=1}^{j=J} u_j d_j \prod_{i=1}^{i=j} (1-p_i) + \sum_{j=1}^{j=J} u_j p_j \tau_j \prod_{i=0}^{i=j-1} (1-p_i)$, indicating the expected quantity produced during the in-control process, $I^{\text{OC}} = \sum_{j=1}^{j=J} u_j p_j (d_j - \tau_j) \prod_{i=0}^{i=j-1} (1-p_i) + b \sum_{j=1}^{j=J-1} u_j p_j E(V_j) \prod_{i=0}^{i=j-1} (1-p_i)$, where $E(V_j) = \sum_{i=j+1}^{i=J} b^{i-j-1} d_i$, indicating the expected quantity produced during the out-of-control process. Qual represents the ratio of good (defect-free) units produced to the total units started. It quantifies losses from defects and rework.

Consequently, the expected OEE can be written as

$$E(\text{oee}) = \text{Avail} \times \text{Perf} \times \text{Qual}. \tag{21}$$

Using the optimal joint policy obtained from equation (17), $E(\text{oee})$ can be calculated easily. The OEE can serve as an effective metric for evaluating production performance. For complex production systems, the proposed OEE serves as a diagnostic tool: a low availability score may indicate unplanned downtime due to inadequate maintenance or equipment failures, while poor performance often points to production rate inefficiencies or process bottlenecks, and subpar quality highlights gaps in quality control (e.g. SPC implementation). Beyond evaluation, OEE provides a data-driven pathway for continuous improvement. By decomposing proposed OEE into its constituent components, Avail, Perf, and Qual, organizations can pinpoint root causes of inefficiency.

4. Numerical analysis

This section uses the 1580 hot rolling production line for thin strip steel to demonstrate the proposed method. Several key procedures in the hot rolling process are: heating, roughing, shearing, finishing, cooling, and coiling, with finishing as the core procedure. As the convexity of strip steel indicates the quality of the shape, it is an important quality characteristic in the finishing process. The convexity is defined as the difference between the thickness at the center of the cross section and the thickness at the reference point situated 40 mm from the edge of the strip steel Deng *et al* (2019), i.e. $h_c - \frac{h+h'}{2}$, where h_c represents the thickness of the center, while h and h' respectively represent the thickness of the left and right reference points, as illustrated in figure 3. The average value of the convexity of strip steel is 40 mm, and the X-bar control chart is used to monitor the mean of this quality characteristic.

Since the production rate significantly influences the deterioration of the finishing mill, operators periodically adjust it to maintain high production efficiency and quality.

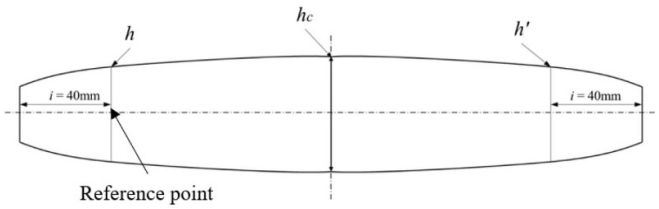


Figure 3. A schematic representation of the convexity of strip steel.

Table 1. Data associated with the finishing procedure.

Parameter	Value	Parameter	Value
$\lambda_{\min}$	0.01	C_{I}	100$
$\lambda_{\max}$	0.5	r	30$
δ	2	γ_0	9$
C_{cm}	800$	γ_1	150$
C_{pm}	200$	T_{cm}	5h
C_{f}	1$	T_{pm}	2h
C_{v}	0.1$	η	2

Table 1 presents the processed production data associated with the finishing procedure. Please note that we process the data prior to use to meet the company's confidentiality requirements.

Using the above information, we aim to obtain the optimal joint policy $\{v_1,\cdots,v_K,l_1,\cdots,l_K,k,n,d_1\}$ for production, quality control, and maintenance.

4.1. Basic analysis

(i) Optimal solution at $\lambda_{\max}=0.5$

We define the unit time Δt as one hour, with a total of 2000 degradation intervals before reaching the failure threshold, resulting in $M=2001$. We use the model (17) and the genetic algorithm for obtaining the optimal solution. Two convergence criteria in the genetic algorithm implemented are defined: First, the algorithm terminates when the fitness value of the optimal solution remains stable within a predefined threshold of 10^{-4} for 50 consecutive generations; Second, the algorithm terminates when the maximum number of iterations reaches 500. We did statistical verification of the convergence performance by running the algorithm more than 30 independent times with different initial random seeds. When retaining four decimal places, the results demonstrate that the algorithm achieves consistent optimal solutions across all runs. The optimal solutions are presented in table 2. Under the optimal setting, $v_1=1$, $l_1=4$, $k=3.3968$, $n=7$, $d_1=132$, the maximum production rate is $E(e)=19.1774$\$, the corresponding OEE is $E(oee)=0.9856$, and the duration of the PM cycle is $T_{\mathrm{c}}=144h$. From the table, it is clear that the system runs at its maximum production rate consistently; therefore, the earnings rate remains the same, regardless of K.

From table 2, we can see that for the same optimal solution, the value of k can differ within a certain range, indicating that the optimal solutions are not sensitive to the coefficient of the control chart.

Table 2. Optimal solution for varying K at $\lambda_{\max}=0.5$.

K	v_1	v_2	v_3	v_4	l_1	l_2	l_3	l_4	k	n	d_1	$E(e)$	T_{c}
1	1				4				3.3968	7	132	19.1774	144
2	1	1			1	3			3.4286	7	132	19.1774	144
3	1	1	1		1	2	1		3.4603	7	132	19.1774	144
4	1	1	1	1	1	1	1	1	3.4333	7	132	19.1774	144

Table 3. Optimal solution for varying K at $\lambda_{\max}=3.5$.

K	v_1	v_2	v_3	v_4	l_1	l_2	l_3	l_4	k	n	d_1	$E(e)$	T_{c}
1	1				8				3.3355	6	10	4.2614	18
2	0.9786	1			1	7			3.3548	6	10	4.2656	18
3	0.9267	0.9800	1		1	1	6		3.3065	6	11	4.2669	19
4	0.9267	0.9800	1	1	1	1	4	2	3.3165	6	11	4.2669	19

Table 4. Optimal solution for varying K at $\lambda_{\max}=4.5$.

K	v_1	v_2	v_3	v_4	l_1	l_2	l_3	l_4	k	n	d_1	$E(e)$	T_{c}
1	0.3331				3				3.4788	6	37	1.3179	43
2	0.3323	0.4032			1	2			3.4032	6	37	1.3231	42
3	0.3323	0.4032	0.4032		1	1	1		3.4226	6	37	1.3231	42
4	0.3323	0.3516	0.3774	0.3452	1	1	1	1	3.5000	6	36	1.3158	43

(ii) Optimal solution at relatively high $\lambda_{\max}$

The production rate is adjustable to control the degradation speed; however, when the degradation rate is low, it may not be necessary to adjust it, and the system can operate at the maximum production rate at all times. In order to observe the change of the production rate under the case of relatively high degradation rate, we increase the maximum degradation rate $\lambda_{\max}$ to 3.5 and 4.5. The corresponding results are shown in tables 3 and 4, respectively.

Table 3 shows that when $K=3$, the earnings rate reaches its maximum of 4.2669, with $v_1=0.9267$, $v_2=0.98$, $v_3=1$, $l_1=1$, $l_2=1$, and $l_3=6$. In comparison to the case of $K=1$, it is clear that adjusting the production rate effectively controls the degradation, leading to an increase in the earnings rate. Additionally, the cases of $K=3$ and $K=4$ are actually the same, indicating that the system only needs to adjust the production rate twice. Notably, as long as the sum of l_3 and l_4 equals 6, the results are optimal for $K=4$.

Table 4 shows that when $K=2$, the earnings rate reaches its maximum of 1.3231, with $v_1=0.3323$, $v_2=0.4032$, $l_1=1$, and $l_2=2$. Since there are the same two sampling intervals in both $K=2$ and $K=3$ for the production rate of 0.4032, they are practically the same. In comparison to the case of $K=1$, it is clear that appropriately adjusting the production rate can effectively control the degradation. The change from $K=3$ to $K=4$ indicates that once the condition that having only one sampling interval for each production rate is satisfied, there is

Table 5. Availability, performance, and quality for OEE calculation.

	$\lambda_{\max} = 3.5$				$\lambda_{\max} = 4.5$			
K	avail	perf	qual	$E(\text{oee})$	avail	perf	qual	$E(\text{oee})$
1	0.8830	1	0.9979	0.8811	0.9522	0.3331	0.9973	0.3163
2	0.8839	0.9881	0.9981	0.8717	0.9510	0.3407	0.9974	0.3232
3	0.8887	0.9555	0.9981	0.8475	0.9510	0.3407	0.9974	0.3232
4	0.8887	0.9555	0.9981	0.8475	0.9519	0.3363	0.9978	0.3195

no need to adjust the production rate; otherwise, the production rate will decrease.

From tables 2–4, it can be observed that a relatively conservative strategy is adopted: when the degradation rate reaches a sufficiently high threshold, the production rate should be reduced. Notably, the production rate in the early operational phase is generally lower than that in the later phase. From the changes of d_1, it can be noted that the sampling frequency increases as the degradation rate accelerates.

Another interesting phenomenon is that the high production rates always have the most sampling intervals. For example, in table 3, for $K = 2$, the production rate of 1 has 7 intervals, while 0.9786 has only 1. Similarly, for $K = 3$, the production rate of 1 has 6 intervals, while both 0.9267 and 0.9800 have only one. The same conclusion can be drawn from table 4.

(iii) Analysis for OEE

The availability, performance, and quality for OEE calculation at $\lambda_{\max}$ of both 3.5 and 4.5 are presented in table 5. It is worth mentioning that a maximum earnings rate may not correspond to a maximum OEE. For example, for $\lambda_{\max} = 3.5$, the maximum earnings rate has the minimum OEE, whereas for $\lambda_{\max} = 4.5$, the maximum earnings rate coincides with the maximum OEE. In practice, if profitability is the priority, earnings rate serves as the key performance indicator (KPI); if the focus is on asset utilization or lean manufacturing, OEE is more appropriate. In the short term, a company may prioritize one objective—even at the expense of the other—based on strategic needs. However, in the long run, these two objectives should align and reinforce each other.

From table 5, we can see that the performance changes the most and has the greatest impact on the OEE, while the availability and quality remain relatively steady. This phenomenon is due to the measure of adjusting the production rate to control the degradation.

Based on the performance values, we can conclude that the average production rate varies significantly with $\lambda_{\max}$. For example, for $K = 1$, it changes from 1 to 1 to 0.3331; for $K = 2$, it changes from 1 to 0.9881 to 0.3407, and so on. This also illustrates that adjusting the production rate to control degradation is preferable.

(iv) Lengths of the sampling intervals

The lengths of the sampling intervals for $\lambda_{\max}$ values of 0.5, 3.5 and 4.5 when $K = 1$ are shown in table 6. It is evident that the initial sampling interval for each value of $\lambda_{\max}$ is considerably longer than subsequent intervals, which reflects a relatively low sampling frequency. This phenomenon can be attributed to the minimal degradation observed in the early stages of the process, where the failure probability remains low and thus frequent sampling is deemed unnecessary.

Table 6. Lengths of the sampling intervals for different $\lambda_{\max}$ values.

$\lambda_{\max}$	v_1	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
0.5	1	132	6	3	3	—	—	—	—
3.5	1	10	2	1	1	1	1	1	1
4.5	0.3331	37	4	2	—	—	—	—	—

Table 7. Effect of the change in δ.

δ	v_1	l_1	k	n	d_1	$E(e)$	T_c	$E(\text{oee})$
1	1	1	3.1575	7	165	19.5126	165	0.9878
2	1	4	3.4055	7	132	19.1774	144	0.9856
3	1	6	3.3622	7	104	18.8017	127	0.9832
4	1	12	3.3543	6	77	18.4040	118	0.9811
5	1	16	3.4409	7	58	18.0201	114	0.9793

Table 8. Effect of the change in η.

η	v_1	l_1	k	n	d_1	$E(e)$	T_c	$E(\text{oee})$
1	1	2	3.0551	19	134	19.1594	140	0.9853
2	1	4	3.4055	7	132	19.1774	144	0.9856
3	1	4	3.7087	4	132	19.1855	144	0.9856
4	1	6	3.7244	2	129	19.1895	144	0.9857
5	1	6	3.937	2	130	19.1911	145	0.9858

4.2. *Sensitivity analysis*

This section exams the effects of the changes in various parameters on the decision variables, several intermediate parameters, as well as the production rate and the OEE.

Table 7 shows the effect of the change in δ. An increasing δ signifies an increased variation magnitude. $E(e)$ increases while $E(\text{oee})$ decreases as expected. To achieve better monitoring performance, the sampling frequency increases by decreasing the duration of the sampling intervals. The duration of the PM cycle T_c also shortens. Since the X-bar chart is effective at monitoring the mean of the quality characteristic, the increase in standard deviation does not significantly affect k and n.

Table 8 shows the effect of the change in the shift coefficient η. As η increases, the shifts become easier to detect; consequently, the sample size significantly decreases, and the duration of the PM cycle slightly increases, even though the sampling interval is slightly shortened. Additionally, the coefficient of the control limits for the X-bar control chart also increases for easily detectable shifts. Both $E(e)$ and $E(\text{oee})$ display an upward trend, indicating that the X-bar control chart is more effective for relatively large shifts.

Table 9. Effect of the change in γ_0.

γ_0	v_1	l_1	k	n	d_1	$E(e)$	T_c	$E(\text{oee})$
$3(0.1r)$	1	4	3.4331	7	132	25.0913	144	0.9856
$9(0.3r)$	1	4	3.4055	7	132	19.1774	144	0.9856
$15(0.5r)$	1	4	3.4567	7	132	13.2636	144	0.9856
$21(0.7r)$	1	3	3.3858	6	131	7.3506	140	0.9854
$27(0.9r)$	1	3	3.3386	6	131	1.4381	140	0.9854

Table 10. Effect of the change in γ_1.

γ_1	v_1	l_1	k	n	d_1	$E(e)$	T_c	$E(\text{oee})$
$150(5r)$	1	4	3.4055	7	132	19.1774	144	0.9856
$300(10r)$	1	4	3.4646	7	129	19.1406	140	0.9855
$600(20r)$	1	5	3.4173	7	125	19.0934	137	0.9853
$900(30r)$	1	5	3.4331	7	122	19.0632	134	0.9851
$3000(100r)$	1	11	3.3937	6	115	18.9494	131	0.9848

Table 11. Effect of the change in C_{pm}.

C_{pm}	v_1	l_1	k	n	d_1	$E(e)$	T_c
100	1	2	3.378	7	130	19.8810	135
150	1	3	3.5039	7	131	19.5240	140
200	1	4	3.4055	7	132	19.1774	144
250	1	4	3.3701	7	134	18.8417	147
300	1	5	3.4094	7	134	18.5153	149

Table 12. Effect of the change in C_{cm}.

C_{cm}	v_1	l_1	k	n	d_1	$E(e)$	T_c
600	1	4	3.5039	7	132	19.1991	144
800	1	4	3.4055	7	132	19.1774	144
1000	1	3	3.4567	7	131	19.1607	140
1200	1	3	3.4803	7	131	19.1459	140
1400	1	2	3.3937	7	132	19.1325	138

Tables 9 and 10 show the effects of the change in the quality loss per unit time produced by both the in-control and out-of-control processes. As γ_0 increases, the sampling frequency, sample size, duration of the PM cycle and OEE exhibit slight changes, while $E(e)$ decreases significantly. As γ_1 increases, the length of sampling intervals decreases, leading to an increase in sampling frequency for a rapid detection for system failures. Additionally, the duration of the PM cycle shortens, allowing earlier maintenance. As expected, both $E(e)$ and $E(\text{oee})$ decrease.

Tables 11 and 12 shows the effects of the change in C_{pm} and C_{cm}, respectively. As expected, as either C_{pm} or C_{cm} increases, $E(e)$ decreases. With an increasing C_{pm}, the sampling frequency decreases, resulting in a longer PM cycle. However, as C_{cm} increases, the sampling frequency remains relatively steady, while the duration of the PM cycle decreases.

Since the production rate remains at its maximum, $E(\text{oee})$ is influenced by either Avail or Qual or both, as shown in table 13.

Table 13. Effect of the change in C_{pm} and C_{cm} on Avail, Qual, and $E(\text{oee})$.

C_{pm}	Avail	Qual	$E(\text{oee})$	C_{cm}	Avail	Qual	$E(\text{oee})$
100	0.9853	0.9998	0.9851	600	0.9860	0.9997	0.9856
150	0.9857	0.9997	0.9854	800	0.9860	0.9997	0.9856
200	0.9860	0.9997	0.9856	1000	0.9857	0.9997	0.9854
250	0.9861	0.9996	0.9857	1200	0.9855	0.9997	0.9854
300	0.9862	0.9995	0.9858	1400	0.9855	0.9997	0.9852

Although Qual exhibits a slight decreasing trend with the increasing C_{pm}, Avail increases in a relatively larger magnitude, resulting an increase in $E(\text{oee})$. Conversely, when C_{cm} increases, Avail decreases significantly while Qual remains constant, resulting in a decrease in $E(\text{oee})$.

5. Conclusion

Our study considers an adjustable production rates in the joint optimization of the X-bar control chart and preventive maintenance, aiming to obtain the optimal joint policy for production, SPC, and maintenance. The assumption of an adjustable production rate aligns better with real-world situations. The stochastic process model is used to describe the system's degradation, accommodating a wider range of potential degradation scenarios. Additionally, the recursive method for deriving various variables related to the production rate enhances the optimization model. Moreover, the availability and quality are defined from the perspective of process states, and the subsequent OEE for the complex production system under study services as an effective indicator for evaluating production performance. This research not only contributes to bridging existing gaps in understanding the interplay between control chart, preventive maintenance, and production, but also provides a practical and comprehensive approach for optimizing their joint policy within an economic framework and analyzing the production performance through the formulation of OEE.

The numerical analysis indicates that selecting an appropriate number of production rates is crucial for optimization. The maximum degradation rate has the greatest influence on the production rate. Additionally, the maximum earnings rate does not necessarily result in a maximum OEE. Other interesting conclusions are also obtained by conducting sensitivity analysis.

With a widespread application of sensors, degradation states can be obtained directly from certain signals or indirectly reflected by other related signals. Therefore, it is worthwhile to combine these signals and quality characteristics to study production optimization. Moreover, in future work, we plan to adopt a multi-objective optimization approach to generate solutions that effectively balance earnings rate and OEE, rather than prioritizing one metric at the expense of the other.

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Appendix

Proof of Proposition1.

Proposition 1. For the above two types of degradation processes, their respective parameters have the following relationships with the production rate.

i) For the gamma process, $\theta = \frac{\lambda_{\max}^2}{\sigma_{\max}^2}$ and $\beta = \lambda(u)\frac{\sigma_{\max}^2}{\lambda_{\max}^2}$,
ii) For the Wiener process, $\mu = \lambda(u)$ and $\sigma = \lambda(u)\frac{\sigma_{\max}}{\lambda_{\max}}$.

Proof of i).

Based on the property i) in the definition 1 that $E(x|u) = \lambda(u)$, we can obtain that $E(x|u=1) = \lambda(1) = \lambda_{\max}$. Based on the property iii) in the definition 1, we can obtain that $\frac{\sqrt{Var(x|u)}}{E(x|u)} = \frac{\sqrt{Var(x|u=1)}}{E(x|u=1)}$. Therefore, given that $E(x|u) = \theta\beta$, $Var(x|u) = \theta\beta^2$, $E(x|u=1) = \lambda_{\max}$, and $E(x|u=1) = \sigma_{\max}^2$, we can derive that $\frac{\sqrt{\theta}\beta}{\theta\beta} = \frac{\sigma_{\max}}{\lambda_{\max}}$, further obtain that $\theta = \frac{\lambda_{\max}^2}{\sigma_{\max}^2}$. Additionally, we have $E(x|u) = \lambda(u) = \theta\beta$. By substituting $\theta = \frac{\lambda_{\max}^2}{\sigma_{\max}^2}$, we ultimately obtain $\beta = \lambda(u)\frac{\sigma_{\max}^2}{\lambda_{\max}^2}$.

Proof of ii).

Similar to the proof of i), it is clear that $\mu = \lambda(u)$ since $E(x|u) = \mu$ and $E(x|u) = \lambda(u)$. Given that $Var(x|u) = \sigma^2$, we can apply the property iii) to derive that $\frac{\sigma}{\mu} = \frac{\sigma_{\max}}{\lambda_{\max}}$. By substituting $\mu = \lambda(u)$, we further obtain $\sigma = \lambda(u)\frac{\sigma_{\max}}{\lambda_{\max}}$.

Proof of Proposition 2.

Proposition 2. The expected duration of a renewal cycle comprises the expected in-control duration, out-of-control duration, downtime duration (CM duration and PM duration), which can be expressed as

$$\begin{aligned} E(T) = &\sum_{j=1}^{j=J} d_j \prod_{i=1}^{i=j}(1-p_i) + \sum_{j=1}^{j=J} p_j\tau_j \prod_{i=0}^{i=j-1}(1-p_i) \\ &+ \sum_{j=1}^{j=J} p_j(d_j-\tau_j)\prod_{i=0}^{i=j-1}(1-p_i) + b\sum_{j=1}^{j=J-1} p_j E(V_j) \\ &\times \prod_{i=0}^{i=j-1}(1-p_i) + \prod_{j=1}^{j=J}(1-p_j)T_{\mathrm{pm}} + T_{\mathrm{cm}}\sum_{j=1}^{j=J} p_j \prod_{i=0}^{i=j-1}(1-p_i) \end{aligned} \tag{12}$$

where $E(V_j) = \sum_{i=j+1}^{i=J} b^{-j-1} d_i$.

Proof.

Since directly expressing $E(T)$ is challenging, we propose a recursive approach to derive it.

Table A1. Duration for the first sampling interval.

Scenario	Probability	Current duration	Expected remaining duration
S_1	$(1-p_1)(1-a)$	d_1	$E(T_1)$
S_2	$(1-p_1)a$	d_1	$E(T_1)$
S_3	$p_1 b$	$\tau_1 + t_1^{\mathrm{OC}}$	$E(V_1) + T_{\mathrm{cm}}$
S_4	$p_1(1-b)$	$\tau_1 + t_1^{\mathrm{OC}}$	T_{cm}

Suppose that $E(T_j)$ is the expected remaining duration beyond time t_j, assuming that the process is in-control at time t_j. $E(V_j) + T_{\mathrm{cm}}$ represents the expected remaining time beyond t_j, assuming that the process is out-of-control at t_j without activating a true alarm. Let τ_j represent the expected in-control duration during the jth sampling interval, and t_j^{OC} represent the out-of-control duration, both under the assumption that the process is out of control at t_j. The details about duration for the first sampling interval are presented in table A1.

Thus, the expected duration of a renewal cycle can be written by

$$\begin{aligned} E(T) = &(1-p_1)(d_1 + E(T_1)) + p_1\left(\tau_1 + t_1^{\mathrm{OC}}\right) \\ &+ p_1 b E(V_1) + p_1 T_{\mathrm{cm}}. \end{aligned} \tag{A.1}$$

For $j = 1, \cdots, J-2$, $E(T_j)$ can be expressed by

$$\begin{aligned} E(T_j) = &(1-p_{j+1})(d_{j+1} + E(T_{j+1})) + p_{j+1}\left(\tau_{j+1} + t_{j+1}^{\mathrm{OC}}\right) \\ &+ p_{j+1} b E(V_{j+1}) + p_{j+1} T_{\mathrm{cm}}. \end{aligned} \tag{A.2}$$

Clearly, $E(T_J) = T_{\mathrm{pm}}$, and $E(T_{J-1}) = (1-p_J)(d_J + T_{\mathrm{pm}}) + p_J\left(\tau_J + t_J^{\mathrm{OC}}\right) + + p_J T_{\mathrm{cm}}$.

By successively substituting $E(T_j)$, $j = 1, \cdots, J-1$, into the equation (A.1), we derive $E(T)$ as

$$\begin{aligned} E(T) = &\sum_{j=1}^{j=J} d_j \prod_{i=1}^{i=j}(1-p_i) + \sum_{j=1}^{j=J} p_j\tau_j \prod_{i=0}^{i=j-1}(1-p_i) \\ &+ \sum_{j=1}^{j=J} p_j t_j^{\mathrm{OC}} \prod_{i=0}^{i=j-1}(1-p_i) + b\sum_{j=1}^{j=J-1} p_j E(V_j) \prod_{i=0}^{i=j-1}(1-p_i) \\ &+ \prod_{j=1}^{j=J}(1-p_j)T_{\mathrm{pm}} + T_{\mathrm{cm}}\sum_{j=1}^{j=J} p_j \prod_{i=0}^{i=j-1}(1-p_i). \end{aligned} \tag{A.3}$$

By replacing with $d_j - \tau_j$, the final form of $E(T)$ is acquired as in the equation (12). Next, we need to derive $E(V_j)$ in the equation (A.3).

Clearly, $E(V_J) = 0$, $E(V_{J-1}) = d_J$, and for $j = 1, \cdots, J-2$,

Table A2. Process control cost.

Scenario	Probability	Current process control cost	Expected remaining process control cost
S_1	$(1-p_1)(1-a)$	$c_f + c_v n$	$E(H_1)$
S_2	$(1-p_1)a$	$c_f + c_v n + C_I$	$E(H_1)$
S_3	$p_1 b$	$c_f + c_v n$	$C_{cm} + (c_f + c_v n)N_1$
S_4	$p_1(1-b)$	$c_f + c_v n$	C_{cm}

$$\begin{aligned} E(V_j) &= d_{j+1} + bE(V_{j+1}) \\ &= d_{j+1} + bd_{j+2} + \cdots + b^{J-j-1}E(V_{J-1}) \\ &= \sum_{i=j+1}^{i=J} b^{i-j-1} d_i \end{aligned} \tag{A.4}$$

Proof of proposition 3.

Proposition 3. The expected process control cost throughout the entire renewal cycle can be written as

$$\begin{aligned} E(H) = {} & (c_f + c_v n) \sum_{j=1}^{j=J-1} \prod_{i=0}^{i=j-1} (1-p_i) + aC_I \sum_{j=1}^{j=J-1} \prod_{i=1}^{i=j} (1-p_i) \\ & + b(c_f + c_v n) \sum_{j=1}^{j=J-1} p_j N_j \prod_{i=0}^{i=j-1} (1-p_i) \\ & + C_{cm} \sum_{j=1}^{j=J} p_j \prod_{i=0}^{i=j-1} (1-p_i) + C_{pm} \prod_{j=1}^{j=J} (1-p_j). \end{aligned} \tag{13}$$

Proof.

The recursive approach used in the proof of Proposition 2 is also used in the derivation of $E(H)$.

Assuming that the process is in-control at t_j, the expected remaining process control cost beyond time t_j is denoted as $E(H_j)$. Thus, for both S_1 and S_2 at t_1, the expected remaining quality control cost is $E(H_1)$. For S_3 at t_1, due to the second type of error, the expected remaining quality control consists of the average cost of a CM and the sampling cost after t_j, i.e. $C_{cm} + (c_f + c_v)N_1$. For S_4 at t_1, the expected remaining quality control is C_{cm}. Regarding the current quality control cost, S_1, S_3 and S_4 include only the sampling cost, while S_2 also encompasses an additional cost for identifying a false alarm, C_I. The details about process control cost for the first sampling interval are presented in table A2.

The expected process control cost over the entire renewal cycle can be expressed as

$$\begin{aligned} E(H) = {} & c_f + c_v n + (1-p_1)aC_I + (1-p_1)E(H_1) \\ & + p_1 b(c_f + c_v n)N_1 + p_1 C_{cm}. \end{aligned} \tag{A.5}$$

For time $t_j, j = 1, \cdots, J-2$, $E(H_j)$ can be expressed by

$$\begin{aligned} E(H_j) = {} & c_f + c_v n + (1-p_{j+1})aC_I + (1-p_{j+1})E(H_{j+1}) \\ & + p_{j+1} b(c_f + c_v n)N_{j+1} + p_{j+1} C_{cm}. \end{aligned} \tag{A.6}$$

Exceptionally, $E(H_{J-1}) = (1-p_J)C_{pm} + p_J C_{cm}$.

By substituting $E(H_j)$, $j = 1, \cdots, J-1$, into the equation (A.5) in turn, we can obtain the $E(H)$ as in the equation (13).

Table A3. Quality loss.

Scenario	Probability	Current quality loss	Expected remaining quality loss
S_1	$(1-p_1)(1-a)$	$\gamma_0 u_1 d_1$	$E(Q_1)$
S_2	$(1-p_1)a$	$\gamma_0 u_1 d_1$	$E(Q_1)$
S_3	$p_1 b$	$\gamma_0 u_1 \tau_1 + \gamma_1 u_1 (d_1 - \tau_1)$	$E(Z_1)$
S_4	$p_1(1-b)$	$\gamma_0 u_1 \tau_1 + \gamma_1 u_1 (d_1 - \tau_1)$	0

Proof of Proposition 4.

Proposition 4. The quality loss produced throughout the entire renewal cycle, denoted as $E(Q)$, can be expressed by

$$\begin{aligned} E(Q) = {} & \gamma_0 \sum_{j=1}^{j=J} u_j d_j \prod_{i=1}^{i=j} (1-p_i) + \gamma_0 \sum_{j=1}^{j=J} p_j u_j \tau_j \prod_{i=0}^{i=j-1} (1-p_i) \\ & + \gamma_1 \sum_{j=1}^{j=J} p_j u_j (d_j - \tau_j) \prod_{i=0}^{i=j-1} (1-p_i) \\ & + b \sum_{j=1}^{j=J-1} p_j E(Z_j) \prod_{i=0}^{i=j-1} (1-p_i) \end{aligned} \tag{14}$$

where $E(Z_j) = \gamma_1 \sum_{i=j+1}^{i=J} b^{i-j-1} u_i d_i$.

Proof.

The recursive approach is used again. Let $E(Q_j)$ represent the expected remaining quality loss beyond time t_j, assuming that the process is in-control at t_j. Additionally, let $E(Z_j)$ represent the expected remaining quality loss beyond t_j, given that the process is out-of-control at t_j without triggering a true alarm. Both S_1 and S_2 at t_1 indicate an in-control process, resulting in a current quality loss of $\gamma_0 u_1 d_1$ and an expected remaining quality loss of $E(Q_1)$. In contrast, S_3 and S_4 means a shift occurs during the sampling interval, leading to a quality loss of for the in-control part and $\gamma_0 u_1 \tau_1$ for the out-of-control part. Depending on whether there is a true alarm, the expected remaining quality loss is $E(Z_1)$ for S_3, and 0 for S_4. The details about quality loss for the first sampling interval are shown in table A3.

The quality loss during the entire renewal cycle can be expressed as

$$\begin{aligned} E(Q) = {} & (1-p_1)\gamma_0 u_1 d_1 + (1-p_1)E(Q_1) + p_1 \gamma_0 u_1 \tau_1 \\ & + p_1 \gamma_1 u_1 (d_1 - \tau_1) + p_1 b E(Z_1). \end{aligned} \tag{A.7}$$

Clearly $E(Q_J) = 0$, $E(Q_{J-1}) = (1-p_J)\gamma_0 u_J d_J + p_J \gamma_0 u_J \tau_J + p_J \gamma_1 u_J (d_J - \tau_J)$, and for $j = 1, \cdots, J-2$, $E(Q_j)$ can be derived as

$$\begin{aligned} E(Q_j) = {} & (1-p_{j+1})\gamma_0 u_{j+1} d_{j+1} + (1-p_{j+1})E(Q_{j+1}) \\ & + p_{j+1} \gamma_0 u_{j+1} \tau_{j+1} + p_{j+1} \gamma_1 u_{j+1} (d_{j+1} - \tau_{j+1}) \\ & + p_{j+1} b E(Z_{j+1}). \end{aligned} \tag{A.8}$$

By successively substituting $E(Q_j)$ for $j = 1, \cdots, J-1$ into the equation (A.7), we can obtain the $E(Q)$ as in the

equation (14). Next, we need to derive the $E(Z_j)$ expression in the $E(Q)$.

Additionally, $E(Z_J)=0, E(Z_{J-1})=\gamma_1 u_J d_J$ and for $j=1,\cdots,J-2$,

$$E(Z_j)=\gamma_1 u_{j+1}d_{j+1}+bE(Z_{j+1})=\gamma_1 u_{j+1}d_{j+1}+b\gamma_1 u_{j+2}d_{j+2} +\cdots+b^{J-j-1}E(Z_{J-1})=\gamma_1\sum_{i=j+1}^{i=J}b^{i-j-1}u_i d_i \tag{A.9}$$

Proof of proposition 5.

Proposition 5. The expected sales income throughout the entire renewal cycle, denoted as $E(R)$, can be expressed by

$$E(R)=r\sum_{j=1}^{j=J}u_j d_j\prod_{i=1}^{i=j}(1-p_i)+r\sum_{j=1}^{j=J}p_j u_j\tau_j\prod_{i=0}^{i=j-1}(1-p_i). \tag{15}$$

Proof.

Let $E(R_j)$ represent the expected remaining sales income beyond time t_j, given that the process is in-control at t_j. As the sales income is defined as the income generated from the in-control process, the expected duration of a renewal cycle can be written as

$$E(R)=(1-p_1)(ru_1d_1+E(R_1))+p_1 ru_1\tau_1. \tag{A.10}$$

Obviously, $E(R_J)=0$, and $E(R_{J-1})=(1-p_J)ru_J d_J+p_J ru_J\tau_J$. For $j=1,\cdots,J-2$, $E(R_j)$ can be expressed by

$$E(R_j)=(1-p_{j+1})(ru_{j+1}d_{j+1}+E(R_{j+1}))+p_{j+1}ru_{j+1}\tau_{j+1}. \tag{A.11}$$

By successively substituting $E(R_j)$ into the equation (A.10), we acquire the $E(R)$ as in the equation (15).

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